1054

Roll No. ....

Printed Pages : 3

GSQ/D-19

MATHEMATICS

Paper–BM-352

Groups and Rings

Time allowed : 3 hours] *[Maximum marks : 40*

***Note** : Attempt five questions in all, selecting one question from each section. Question No. 1 is compulsory.*

(Compulsory Question)

1. (a) Prove that an arbitrary intersection of subgroups of a group is a subgroup. 2

(b) Define Inner automorphism with an example. 1½

(c) Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$ as the product of disjoint cycles. 1½

(d) Prove that a field has no proper ideals. 2

(e) Define Euclidean Ring. 1

Section–I

2. (a) If a group has four elements, then show that it must be abelian. 4

(b) Prove that every subgroup of a cyclic group is cyclic. 4

3. (a) If an abelian group of order 6 contains an element of order 3, show that it must be a cyclic group. 4

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(b) Prove that order of every element of a finite group is a divisor of the order of the group. 4

Section–II

4. (a) The set Inn (G) of all inner automorphism of a group G is isomorphic to the quotient group G/Z (G), where Z (G) is the centre of a group G, i.e. $\text{Inn}(G) \cong G/Z(G)$ 4

(b) Let G' be the commutative subgroup of a group G. Then G is abelian iff $G' = \{e\}$, where e is the identity element of G. 4

5. (a) If H and K are subgroups of a group G and H is normal in G, then $HK/H \cong K/(H \cap K)$. 4

(b) Define Kernel of Homomorphism $f : G \to G'$, and prove that Kernel of f is a normal subgroup of G. 4

Section–III

6. (a) Prove that an ideal of a ring of integers is maximal iff it is generated by some prime integers. 4

(b) Let f be a ring isomorphism of R onto R'. Show that if R is an Integral domain, then R' is also an integral domain. 4

7. (a) Let R be a ring with unity element such that R has no ideals except {0} and R. Prove that R is a division ring. 4

(b) Show that S is an ideal of S + T, where S is any ideal of a ring R and T is any subring of R. 4

Section–IV

8. (a) Prove that the ring of Gaussian integers is an Euclidean domain (ring) 4

(b) Show that $\sqrt{-5}$ is a prime element of the ring $Z\sqrt{-5} = \{a + \sqrt{-5}\,b\ ;\ a, \in Z\}$. 4

9. (a) If F is a field, then prove that F (x) is an Euclidean domain. 4

(b) Show that the polynomial $x^3 + 3x + 1$ is irreducible over Q. 4