

Roll No.

Printed Pages : 3

BAE / A-18
MATHEMATICS
Paper-BM-102
Calculus

*Time allowed : 3 hours]**[Maximum marks : 26]*

Note :- Attempt five questions, selecting at least one question from each Section.

Section-I

1. (a) Show that \sqrt{x} is a continuous function for $x \geq 0$. 2½
- (b) Expand $\tan x$ in powers of $\left(x - \frac{\pi}{4}\right)$ up to first four terms. 2½
2. (a) Find the asymptotes of $r^n f_n(\theta) + r^{n-1} f_{n-1}(\theta) + \dots + f_0(\theta) = 0$
- (b) If $y = (\sin^{-1}x)^2$, prove that

$$(1-x^2)y_{n+2} - (2n+1)y_{n+1}x - n^2.y_n = 0$$
 2½
3. (a) Find the radius of curvature at the origin for the curve.

$$2x^4 + 4x^3y + xy^2 + 6y^3 - 3x^2 - 2xy + y^2 - 4x = 0.$$
 2½
- (b) Trace the curve $r = a(1+\cos\theta)$ 2½

Section-II

4. (a) Prove that $\int_0^{\frac{\pi}{2}} \sin^{2n} x dx = \frac{12n}{(2^n \ln 2)^2} \cdot \frac{\pi}{2}$ 2½

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(2)

- (b) The circle $x^2 + y^2 = a^2$ is revolved about the x-axis. Find the volume of the sphere so formed. 2½

5. (a) Show that the volume generated by the revolution of the

$$\text{ellipse } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ about the line } x = 2a \text{ is } 4\pi a^2 b. \quad 2\frac{1}{2}$$

- (b) Find the intrinsic equation of the curve $y = a \log \sec \frac{x}{a}$, s being measured from the origin. 2½

6. (a) Find the area of the loop of the curve $x = a(1-t^2)$, $y = at(1-t^2)$ 2½

- (b) Find the length of the complete cycloid given by

$$x = a(\theta + \sin \theta), y = a(1 + \cos \theta)$$
 2½

Section-III

7. (a) Solve the differential equation

$$\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0 \quad 2\frac{1}{2}$$

- (b) Solve the differential equation 2½

$$\frac{d^4y}{dx^4} + a^4y = 0$$

8. (a) Solve the differential equation $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$ 2½

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Turn over

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- (b) Solve the differential equation. 2½

$$(x+a)\frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$$

Section-IV

9. (a) Solve

$$\sin^2 x \cdot \frac{d^2y}{dx^2} = 2y, \text{ given that } y = \cot x \text{ is a solution.} \quad 3$$

- (b) Apply method of variation of parameters to solve

$$\frac{d^2y}{dx^2} + 4y = \tan 2x \quad 3$$

10. (a) Discuss the method for finding particular integral of

$$\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Q.y = 0 \quad 3$$

- (b) Solve $\frac{d^4y}{dx^4} - y = x \sin x.$ 3