

Roll No.

Total Pages : 04

BT-2/M-18

32030

APPLIED MATHEMATICS-II

AS-104-N

(Time : Three Hours)

[Maximum Marks : 75]

Note : Attempt Five questions in all, selecting at least one question from each Unit. All questions carry equal marks.

Unit I

1. (a) Solve the equation $x^4 - 8x^3 + 21x^2 - 20x + 5 = 0$ given that the sum of two of the roots is equal to the sum of the other two. 7.5

- (b) Solve the equation : 7.5

$$8x^5 - 22x^4 - 55x^3 + 55x^2 + 22x - 8 = 0$$

2. (a) State and prove the relation between beta and gamma functions. 7.5

- (b) Using the method of differentiation under the integral sign, evaluate : 7.5

$$\int_0^\pi \frac{\log(1+a\cos x)}{\cos x} dx = \pi \sin^{-1} a$$

Unit II

3. (a) Solve :

(i) $L(t \sin at)$

(ii) $L^{-1}\left(\frac{5s+3}{(s-1)(s^2+2s+5)}\right)$

3+4.5

- (b) Only state the following properties of the Laplace transform : 7.5

- (i) First shifting property
- (ii) Multiplication property
- (iii) Division property
- (iv) Derivative property
- (v) Integral property.

4. (a) State and prove the Convolution theorem and evaluate : 7.5

$$L^{-1}\left(\frac{1}{(s^2+1)(s^2+25)}\right)$$

- (b) Using Laplace transform, solve the simultaneous equations $\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$, given that $x(0) = y(0) = 0$. 7.5

Unit III

5. (a) Solve the differential equation : 7.5

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$$

- (b) Find the orthogonal trajectories of the family of curves : 7.5

$$r = 2a(\cos\theta + \sin\theta)$$

6. (a) By variation of parameter, find the particular integral of equation : 7.5

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin(e^x)$$

- (b) Solve the differential equation : 7.5

$$(D^2 - 1)y = x \sin n$$

Unit IV

7. (a) Find the directional derivative of

$$\phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x, \text{ at the point } P(1, 1, 1) \text{ in}$$

the direction of $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$ 7.5

- (b) Give the geometrical interpretation of Divergence of a vector field. 7.5

8. (a) Evaluate by Green's theorem

$$\iint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$$

where C is the boundary of the region bounded

$$x = 0, y = 0, x + y = 1. \quad 7.5$$

- (b) Using Gauss divergence theorem, evaluate : 7.5

$$\iint_S \bar{F} \cdot \hat{n} ds$$

$$\text{where } \bar{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k} \text{ and}$$

S is the surface of the cube bounded by the planes

$$x = 0, x = a, y = 0, y = b, z = 0, z = c.$$