

Roll No. ....

Total Pages : 04

BT-3/D-19

33030

MATHEMATICS

MATH-201E (Opt. ii)

Mech. Engg. Auto

Time : Three Hours]

[Maximum Marks : 100

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

**Unit I**

1. (a) Find the Fourier series representing  $f(x) = \pi - x$ , for  $0 < x < 2\pi$ . 10

- (b) Find Fourier half-range even expansion of the function,  $f(x) = \left(\frac{-x}{l} + 1\right)$ , for  $0 \leq x \leq l$ . 10

2. (a) Find the Fourier Sine Transform of  $g(x) = e^{-x}$ . 10  
 (b) Using Parseval's identity, prove that :

$$\int_0^{\infty} \frac{dx}{(2^2 + x^2)(3^2 + x^2)} = \frac{\pi}{60}. \quad \text{10}$$

**Unit II**

3. (a) Separate into real and imaginary parts  $\tan^{-1}(x+iy)$ . 10
- (b) Prove that the function  $\cosh z$  is analytic and find its derivative. 10

4. (a) Find the analytic function  $w = u + iv$ , if  $v = \log(x^2 + y^2) + x - 2y$ . 10
- (b) Find the image of  $|z - 2i| = 2$  under the mapping of  $w = \frac{1}{z}$ . 10

**Unit III**

5. (a) X is a continuous random variable with probability density function given by :

$$f(x) = \begin{cases} kx & (0 \leq x < 2) \\ 2k & (2 \leq x < 4), \text{ find } k \text{ and} \\ -kx + 6k & (4 \leq x < 6) \end{cases}$$

mean value of X. 10

- (b) In a bolt factory, machines A, B and C manufacturing 40%, 35% and 25%, of the total. Of their output 2%, 4% and 5% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B or C ?

10

6. (a) If  $x$  is a Poisson variate such that  $P(x = 1)$  is equal to  $P(x = 2)$ . Find the value of the mean of the distribution. 10
- (b) Find the probability of 5 successes in a binomial distribution whose mean and variance are respectively 6 and 2. 10

#### Unit IV

7. Using Simplex method :

$$\text{Max. } Z = 5x + 3y$$

Subject to the constraints

$$x + y \leq 2$$

$$5x + 2y \leq 10$$

$$3x + 8y \leq 12$$

$$x, y \geq 0$$

20

8. Using Dual Simplex method :  
 $\text{Max. } Z = -3x - 2y$   
 subject to the constraints

$$x + y \geq 1$$

$$x + y \leq 7$$

$$x + 2y \geq 10$$

$$y \leq 3$$

$$x, y \geq 0$$

20