

BT-4/M-19

34091

MATHEMATICS-III

Paper : AS-201N

Opt. (ii)

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *five* questions in all. Selecting at least *one* question from each unit. All questions carry equal marks.

UNIT-I

1. (a) Find the Fourier series for the function

$$f(x) = \begin{cases} -1 & \text{for } -\pi < x < -\frac{\pi}{2} \\ 0 & \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1 & \text{for } \frac{\pi}{2} < x < \pi. \end{cases}$$

Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

(7½)

- (b) Obtain the half-range sine-series for e^x in $0 < x < 1$.
(7½)

2. (a) Find the Fourier cosine transform of e^{-x^2} . (7½)

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(b) Using Parseval's identity, Prove that

(7½)

$$\int_0^{\infty} \frac{1 - \cos 2x}{x^2} dx = \pi$$

UNIT-II

3. Solve the following LPP by simplex method

Minimize $Z = x - 3y + 3z$

Subject to $3x - y + 2z \leq 7$; $2x + 4y \geq -12$;

$-4x + 3y + 8z \leq 10$; $x, y, z \geq 0$.

(15)

4. (a) Explain the terms :

(i) Feasible solution.

(ii) Unbounded solution.

(iii) Convex region.

(2½×3)

(b) Using Graphical method :

Maximize $Z = x + 2y$

Subject to $2x + y \leq 8$; $2x + 3y \leq 12$;

$x, y \geq 0$.

(7½)

UNIT-III

5. (a) If $a + ib = \tanh \left(v + i\frac{\pi}{4} \right)$, prove that $a^2 + b^2 = 1$.

(7½)

(b) Show that the function $v(x, y) = \ln(x^2 + y^2) + x - 2y$ is Harmonic. Find its conjugate harmonic function $u(x, y)$ and the corresponding analytic function $f(z)$.

(7½)

6. (a) Evaluate, using Cauchy's integral formula :

$$\int_C \frac{z}{z^2 - 3z + 2} dz, \text{ where } C \text{ is } |z - 2| = \frac{1}{2}. \quad (7\frac{1}{2})$$

- (b) Evaluate $\int_{1-i}^{2+3i} (z^2 + z) dz$ along the line joining the points (1, -1) and (2, 3). (7\frac{1}{2})

UNIT-IV

7. (a) A man is known to speak lie 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. (7\frac{1}{2})
- (b) In a normal distribution 17% of the items are below 30 and 17% of the items are above 60. Find the mean and standard deviations. (7\frac{1}{2})

8. (a) X is a continuous random variable with Probability density function given by

$$f(x) = \begin{cases} kx & \text{if } 0 \leq x < 2 \\ 2k & \text{if } 2 \leq x < 4 \\ -kx + 6k & \text{if } 4 \leq x < 6 \end{cases}$$

Find k and mean value of X . (7\frac{1}{2})

- (b) In 100 sets of ten tosses of an unbiased coin, in how many cases do you expect to get
- (i) 7 heads and 1 tails.
 - (ii) at least 7 heads. (7½)
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