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STATISTICAL MODELS FOR COMPUTER SCIENCE

Paper–CSE–405

Time allowed : 3 hours] *[Maximum marks : 100*

Note : Attempt any five questions, with at least one from each section.

Section–A

1. (a) Two dices are thrown. Let A be the event that sum of faces is odd. B is the event that at least one of the dice shows number 6. Describe the events $A \cup B, A \cap B, A \cap \overline{B}$. Find the probabilities of the above events assuming all outcomes have equal probabilities. 7

   (b) An experiment consists of rolling a die until a 6 is obtained.

      (i) Find the sample space S, if we are interested in all possibilities.

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      (ii) Find the sample space S, if we are interested in the number of throws needed to get a 6. 6

   (c) Let A and B be events in a sample space S. Show that if A and B are independent, then so are (a) A and $\overline{B}$, (b) $\overline{A}$ and B, and (c) $\overline{A}$ and $\overline{B}$. 7

2. (a) For any three events A_1, A_2, and A_3, show that

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$ 10

   (b) An urn contains red, blue, green, yellow, white, and black balls. In how many ways can we choose :

      (i) 10 balls ?

      (ii) 20 balls with least 2 colors each ?

      (iii) 20 balls with no more than 3 blue balls ?

(iv) 20 balls with at least 4 red balls, at least 2 green balls, at least 6 white balls, and no more than 3 brown balls ? 10

Section–B

3. (a) Consider the function given by

$$F(x) = \begin{cases} 0 & x < 0 \\ x + \frac{1}{2} & 0 \le x < \frac{1}{2} \\ 1 & x \ge \frac{1}{2} \end{cases}$$

(i) Sketch F (x) and show that F (x) has the properties of a cdf

(ii) If X is the r.v. whose cdf is given by F (x), find (i) $P(X \le 1/4)$, (ii) $P(0 < X \le 1/4)$, (iii) $P(X = 0)$, and (iv) $P(0 \le X \le 1/4)$.

(iii) Specify the type of X. 10

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(b) Consider a function

$$f(x) = \frac{1}{\sqrt{\pi}} e^{(-x^2 + x - a)} \quad -\infty < X < \infty$$

Find the value of 'a' such that f (x) is a pdf of a continuous r.v. X. 10

4. (a) Assume that the length of a phone call in minutes is an exponential r.v. X with parameter $\lambda = 1/10$. If someone arrives at a phone booth just before you arrive, find the probability that you will have to wait (i) less than 5 minutes, and (ii) between 5 and 10 minutes. 10

(b) Let $Y = aX + b$. Determine the pdf of Y, if X is a uniform r.v. over (0, 1). 10

Section–C

5. (a) Suppose that a random process X (t) is wide-sense stationary with autocorrelation

$$R_x(t, t + \tau) = e^{-|\tau|/2}$$

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(i) Find the second moment of the r.v. X (5).

(ii) Find the second moment of the r.v. X (5) – X (3). 10

(b) Let X (t) be a Poisson process with rate A. Find E $[X^2(t)]$. 10

6. (a) Let X (t) be a stationary random process with zero mean and autocorrelation $R_x(\tau)$. We form the process Y (t), as follows : Y (t) = X (t) + f (t) where f (t) is a known deterministic signal. Find the mean $m_y(t)$ and the autocorrelation $R_y(t_1, t_2)$ of the process Y (t). 10

(b) If x (t) is a normal process with zero mean and $y(t) = e^{ax(t)}$ then find mean n_y and $R_y(\tau)$ 10

Section–D

7. (a) People arrive at a telephone booth according to a Poisson process at an average rate of 12 per hour, and the average time for each call is an exponential r.v. with mean 2 minutes.

(i) What is the probability that an arriving customer will find the telephone booth occupied ?

(ii) It is the policy of the telephone company to install additional booths if customers wait an average of 3 or more minutes for the phone. Find the average arrival rate needed to justify a second booth. 10

(b) Discuss *M/M/1* queuing system in detail and find the probability of queuing. 10

8. (a) A corporate computing centre has two computers of the same capacity. The jobs arriving at the centre are of two types, internal jobs and external jobs. These jobs have Poisson arrival times with rates 18 and 15 per hour, respectively. The service time for a job is an exponetial r.v. with mean 3 minutes.

(i) Find the average waiting time per job when one computer is used exclusively for internal jobs and the other for external jobs.

(ii) Find the average waiting time per job when two computers handle both types of jobs.

(b) In a university computer centre, 80 jobs an hour are submitted on the average. Assuming that the computer service is modeled as an M/M/1 queueing system, what should the service rate be if the average turnaround time (time at submission to time of getting job back) is to be less than 10 minutes ? 10