Roll No. ....

Total Pages : 4

GSM/M-20 **1579**

MATHEMATICS

(Sequences & Series)

Paper–BM–241

Time Allowed : 3 Hours] [Maximum Marks : 27

Note : Attempt **five** questions in all, selecting at least **one** question from each Unit. Question No. **1** is compulsory.

Compulsory Question

1. (a) Define Interior point of a set. 1

(b) Define Limit Point of a set. 1

(c) Give an example of an unbounded set which has no limit point. 1

(d) State Cauchy's root test. 1

(e) Define Null sequence. 1

UNIT–I

2. (a) Prove that the interior of set A is the largest open subject of A. 3

(b) The intersection of an arbitrary family of closed sets is closed. 2½

3. (a) Prove that derived set of any set is a closed set. 3

(b) Prove or disprove $A^{\circ} \cup B^{\circ} = (A \cup B)^{\circ}$. 2½

UNIT–II

4. (a) Show that

$$\lim_{n \to \infty} \left(\frac{1}{n^2} + \frac{1}{(n+1)^2} + \dots + \frac{1}{(2n)^2} \right) = 0.$$ 3

(b) Examine the convergence of the series

$$\sum_{n=3}^{\infty} x^{\log n}.$$ 2½

5. (a) State and prove Cauchy's second theorem on limits. 3

(b) Show that $\lim_{n \to \infty} \left(\frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot - \frac{n}{n-1} \right)^{\frac{1}{n}} = 1.$ 2½

UNIT–III

6. (a) State and prove Cauchy's root test. 3

(b) Test the convergence of the series

$$1+\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+\ldots\ldots., x>0.$$ 2½

7. (a) State and prove D'Alembert's ratio test. 3

(b) Discuss the convergence of the series

$$\frac{1}{2}+\left(\frac{2}{3}\right)x+\left(\frac{3}{4}\right)^2 x^2+\left(\frac{4}{5}\right)^3 x^3+\ldots\ldots..$$ 2½

UNIT–IV

8. (a) Show that

$$\sum_{n=1}^{\infty} \frac{1}{n^p}\cdot a_n \, (P \geq 0)$$

is convergent if $\sum_{n=1}^{\infty} a_n$ is convergent. 3

(b) The product

$$\prod_{n=1}^{\infty} (1+a_n)$$

is absolutely convergent if and only

$$\sum_{n=1}^{\infty} a_n$$

is absolutely convergent. 2½

9. (a) Show that the series

$$\frac{\log^2}{2^2} - \frac{\log^3}{3^2} + \frac{\log^4}{4^2} - \ldots\ldots$$

converges. 3

(b) Show that the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\left(1+\frac{1}{n}\right)^{-n}$$

is convergent. 2½