Roll No. .....

Total Pages : 3

GSM/M-22

1612

SEQUENCE AND SERIES

Paper–BM-241

Time Allowed : 3 Hours]

[Maximum Marks : 40

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) Define limit point of a set and give an example of a set which has three limit points?

   (b) Give an example of a sequence which is bounded but not convergent.

   (c) Discuss the convergence of the series :

$$\sum_{n=1}^{\infty} \sin\frac{1}{n}.$$

   (d) Test the absolute convergence of the infinite series :

$$\sum_{n=1}^{\infty} (-1)^{n-1}\frac{1}{n^2}.$$

2×4=8

UNIT–I

2. (a) Define open set. Prove that arbitrary union of open sets is an open set. 4

   (b) Prove that the set of rational [illegible] complete. 4

3. (a) Define closure of a set. Prove that closure of a set is a closed set. 4

   (b) Show that a set having finite number of elements is compact. 4

UNIT–II

4. (a) State and prove Cauchy's first theorem on limits. 4

   (b) Discuss the convergence of the sequence $\langle a_n \rangle$ where

$$a_n = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}.....+\frac{1}{n}.$$ 4

5. (a) Discuss the convergence of the series :

$$\sum_{n=1}^{\infty} \frac{x^n}{x^n + a^n}, x > 0.$$ 4

   (b) Discuss the convergence of the series : 4

$$\frac{\sqrt{3}}{1.2}+\frac{\sqrt{5}}{3.4}+\frac{\sqrt{7}}{5.6}+.....$$

UNIT–III

6. (a) State and prove Raabe's test for the convergence of an infinite series. 4

   (b) Test the convergence of infinite series : 4

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{2n!}x^n, x > 0.$$

7. (a) State and prove Cauchy Condensation test for the convergence of an infinite series. 4

(b) Test the convergence of $\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$. 4

UNIT–IV

8. (a) Discuss the absolute convergence of the series :

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{n}} (\cos nx)^2$, x is real. 4

(b) Discuss the convergence and absolute convergence of:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\operatorname{cosec}\left(\frac{1}{n}\right)}.$$ 4

9. (a) Test the convergence of $\sum_{n=1}^{\infty} \frac{\cos nx}{n^p}, p > 0.$ 4

(b) Test the convergence of infinite product $\prod_{n=0}^{\infty} [1 - \frac{1}{n^2}]$. 4