Roll No. .... Total Pages : 3

BAE/A-20 **40**

MATHEMATICS

[Vector Analysis and Geometry (For Pvt. only)]

Paper : BM-103

Time : Three Hours] [Maximum Marks : 30

Note : Attempt *five* questions in all, selecting at least *one* question from each unit.

UNIT–I

1. (a) The vector $\overrightarrow{OP} = 5\vec{i} + 12\vec{j} + 13\vec{k}$ turns through an angle $\frac{\pi}{2}$ about O passing through the positive side of y-axis on its way. Find the vector in the new position. 3

(b) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^2 = 4$ at the point $(1, -1, 2)$. 3

2. (a) Evaluate div $\left(\frac{\vec{r}}{r}\right)$ when $|\vec{r}| = r$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

Hence, show that grad div $\left(\frac{\vec{r}}{r}\right) = -\frac{2}{r^3}\vec{r}$. 3

(b) Prove that $\nabla^2\left(\frac{x}{r^2}\right) = -\frac{2x}{r^4}$ where $r = \sqrt{x^2 + y^2 + z^2}$.

3

UNIT–II

3. (a) If $\vec{r} = 2t\vec{i} + 3t^2\vec{j} - t^3\vec{k}$, find $\int_1^2 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2} \right) dt$. 3

(b) If $\vec{f} = (2x^2 - 3z)\vec{i} - 2xy\vec{j} - 4z\vec{k}$, find $\iiint_V (\nabla \times \vec{f})\, dV$

where V is the region bounded by the co-ordinate planes and the plane $2x + 2y + z = 4$. 3

4. Verify Green's theorem in plane for $\oint_C [x^2 + y^2 dx - 2xy\, dy]$,

where C is the rectangle in xy-plane bounded by $y = 0$, $x = a$, $y = b$, $x = 0$. 6

UNIT–III

5. Trace the conic $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$.
6

6. (a) Find the conic confocal with $x^2 + 2y^2 = 2$ which passes through (1, 1). 3

(b) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact. 3

7. (a) Find the equation to the right circular cone whose vertex is P(2, –3, 5), axis PQ which makes equal angles with the axes and which passes through A(1, –2, 3). 3

(b) Find the equation of the right circular cylinder whose guiding circle is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. 3

UNIT–IV

8. (a) Prove that six normals can be drawn from a given point to the ellipsoid. 3

(b) Show that the plane $2x - 4y - z = 3$ touches the paraboloid $x^2 - 2y^2 = 3z$, and also find the point of contact. 3

9. (a) Find the equations of the generating lines of the hyperboloid $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$ which pass through the point $\left(2, -1, \frac{4}{3}\right)$. 3

(b) Prove that the sum of the square of the reciprocals of any three mutually perpendicular diameters of an ellipsoid is constant. 3

10. Prove that

$$x^2 + y^2 + z^3 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$$

represents a paraboloid of revolution and find the equation of axis and the co-ordinates of focus. 6