

Roll No. ....

Total Pages : 04

BCA/M-20

1889

MATHEMATICAL FOUNDATION-II

BCA-123

Time : Three Hours]

[Maximum Marks : 80

**Note :** Attempt *Five* questions in all. Q. No. **1** is compulsory.  
Attempt *four* more questions selecting exactly *one*  
question from each Unit. All questions carry equal  
marks.

**(Compulsory Question)**

1. Explain the following :

- (a) Logical statement
- (b) Universal quantifier
- (c) Binary operation
- (d) Isomorphism of a ring
- (e) Matrix
- (f) Row echelon matrix
- (g) Characteristic equation
- (h) Hermitian matrix.

**8×2=16**

**(3)L-1889**

**1**

**Unit I**

2. (a) If  $p$  and  $q$  be any statements, then construct the truth table of  $\sim p \wedge q$  and  $(p \rightarrow q) \rightarrow (p \wedge q)$ . **8**
- (b) By using laws of algebra of logical statements, prove that  $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$  and  $(p \vee q) \wedge \sim p \equiv \sim p \wedge q$ . **8**
3. (a) Prove by the principle of mathematical induction that the sum of first  $n$  natural numbers  $= n(n + 1)/2$  i.e.,  $1 + 2 + 3 + \dots + n = n(n + 1)/2$ , for all  $n \in \mathbb{N}$ . **8**
- (b) Using principle of mathematical induction, prove that for all  $n \in \mathbb{N}$ ,  $(2n + 7) < (n + 3)^2$ . **8**

**Unit II**

4. (a) Show that  $\mathbb{I}$  (that set of all integers) is an abelian group w.r.t. addition. **8**
- (b) Prove that group  $G$  is abelian iff  $(ab)^2 = a^2b^2$  for all  $a, b \in G$ . **8**
5. (a) Prove that the set of all  $n \times n$  matrices form a ring with identity but it is not commutative ring with respect to matrix addition and multiplication. **8**

- (b) Prove that the intersection of two subrings in a ring. **8**

**Unit III**

6. (a) If  $B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & -1 & 5 \end{bmatrix}$ ,  $C = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ ; find  $3B + 4C$ . **8**

- (b) Without using the concept of inverse of matrix,

find the matrix  $\begin{bmatrix} x & y \\ z & u \end{bmatrix}$  such that

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}. \quad \mathbf{8}$$

7. (a) Find the rank of the matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ . **8**

- (b) Solve the following system of homogeneous equations :

$$3x + 2y + 7z = 0$$

$$4x - 3y - 2z = 0$$

$$5x + 9y + 23z = 0 \quad \mathbf{8}$$

**Unit IV**

8. (a) Find the characteristic roots and spectrum of matrix,

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}. \quad \mathbf{8}$$

- (b) If  $\alpha$  is an eigen value of a non-singular matrix A, then prove that  $\frac{|A|}{\alpha}$  is an eigen value of adj. A. **8**

9. State and prove Cayley-Hamilton theorem and verify

Cayley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ .

**16**