

Roll No.

Total Pages : 4

BCA/M-14

1084

MATHEMATICAL FOUNDATION-IV

Paper-BCA-246

Time Allowed : 3 Hours]

[Maximum Marks : 80

Note : Attempt five questions in all, selecting at least one question from each Unit. Question No. 9 is compulsory. All questions carry equal marks.

UNIT-I

1. (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = \frac{-9}{(x+y+z)^2}$$

- (b) Examine for maximum and minimum values of the function

$$xy + \frac{a^3}{x} + \frac{b^3}{y}, a > 0, b > 0. \quad 8,8$$

2. (a) If u and v are functions of x and y defined by $x=u+e^{-v}\sin u$ and $y=v+e^{-v}\cos u$, find

$$\frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}.$$

- (b) Find the minimum value of the function $x^2 + y^2 + z^2$ subject to the condition $x+y+z=3a$. 8,8

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UNIT-II

3. (a) Find the intrinsic equation of the curve $p = r \sin \alpha$.

- (b) Find the whole length of the curve

$$x^{2/3} + y^{2/3} = a^{2/3}.$$

8,8

4. (a) If $u_n = \int_0^{\pi/2} \theta \sin^n \theta d\theta (n > 1)$; prove that :

$$u_n = \frac{n-1}{n} u_{n-2} + \frac{1}{n^2}$$

and deduce that $u_5 = \frac{149}{225}$.

- (b) Rectify the loop of the curve $3ax^2 = y(y-a)^2$. 8,8

UNIT-III

5. (a) Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$.

- (b) Find the surface area of a sphere of radius 5. 8,8

6. (a) Find the common area to the parabola $y^2 = 4ax$ and $x^2 = 4ay$.

- (b) Evaluate :

$$\iiint (z^5 + z) dx dy dz \text{ over } x^2 + y^2 + z^2 \leq 1. \quad 8,8$$

UNIT-IV

7. (a) If $\alpha^2 < 1$, prove that :

$$\int_0^{\pi/2} \log(1 - \alpha^2 \cos^2 \theta) d\theta = \pi \log\left(\frac{1 + \sqrt{1 - \alpha^2}}{2}\right)$$

- (b) Prove that :

$$\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m). \quad 8,8$$

8. (a) Find the equation of right circular cylinder, whose guiding circle is $x^2 + y^2 + z^2 = 4$; $x + y + z = 3$.
 (b) Find the centre of the two spheres, which touch the plane $x + 2y + 2z - 5 = 0$ at the point $(1, 1, 1)$ and the sphere $x^2 + y^2 + z^2 + 2x + 4y + 6z - 11 = 0$. 8,8

UNIT-V

9. (a) Prove that :

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$$

- (b) Define Beta and Gamma functions.
 (c) Evaluate :

$$\iint_{00}^{11} e^{x+y} dx dy.$$

- (d) If $u = x(1-y)$ and $v = xy$, find $\frac{\partial(x,y)}{\partial(u,v)}$.

- (e) If $u = xy$ $f(y/x)$, show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial v}{\partial y} = 2u.$$

- (f) Find the centre of the section of the sphere $x^2 + y^2 + z^2 = 49$ by the plane $2x + 3y + 6z = 14$.

- (g) Show that the two spheres

$$x^2 + y^2 + z^2 - 4x + 6y + 4 = 0 \text{ and}$$

$$x^2 + y^2 + z^2 + 7x + 10y - 5z + 12 = 0$$

cut orthogonally.

- (h) Find the equation to the cylinder with generator parallel to z-axis and passing through the curve

$$x^2 + y^2 + 2z^2 = 12 \text{ and}$$

$$x + y + z = 1.$$

2×8=16