

Roll No.

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BCAR/M-15

1699

MATHEMATICAL FOUNDATION-IV

Paper-BCA-246

Time Allowed : 3 Hours] [Maximum Marks : 80

Note : Attempt five questions in all, selecting at least one question from each Unit. Question No. 1 is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$

- (b) Evaluate

$$\int_0^{\pi/2} \sin^6 \theta d\theta.$$

- (c) If $u = xyz$, $v = xy + yz + zx$, $w = x + y + z$; verify that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = (x-y)(y-z)(z-x)$$

- (d) Show that :

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$$

4,4,4,4

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P. T. O.

2. (a) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u (1 - 4 \sin^2 u)$$

- (b) Show that :

$$\frac{\partial}{\partial y} \frac{\partial \phi}{\partial z} \frac{\partial z}{\partial x} = \frac{\partial}{\partial x} \frac{\partial \phi}{\partial y}$$

where $f(x, y) = 0$, $\phi(z, y) = 0$.

8,8

3. (a) Examine for maximum and minimum values the function

$$\sin x + \sin y + \sin(x+y).$$

- (b) Find the minimum value of the function
 $u^2 + v^2 + w^2$ subject to condition

$$au + bv + cw = a + b + c.$$

8,8

UNIT-II

4. (a) If $U_n = \int_0^{\pi/2} \theta \sin^n \theta d\theta$ ($n > 1$); prove that

$$U_n = \frac{n-1}{n} U_{n-2} + \frac{1}{n^2} \text{ and deduce that } U_5 = \frac{149}{225}.$$

(b) Find a reduction formula for $\int x^n \sqrt{2ax - x^2} dx$ and

evaluate $\int_0^{2a} x^3 \sqrt{2ax - x^2} dx.$ 8,8

5. (a) Show that the length of the loop of the curve

$3ay^2 = x(x-a)^2$ is $\frac{4a}{\sqrt{3}}.$

(b) Find the intrinsic equation of the curve whose pedal equation is $p^2 = r^2 - a^2.$ 8,8

UNIT-III

6. (a) Find the area common to the circle $x^2 + y^2 = 4$ and the ellipse $x^2 + 4y^2 = 9.$

(b) Find the surface area of a sphere of radius a. 8,8

7. (a) Evaluate

$$\iint_{x^2+y^2 \leq 1} x^2 y^2 dx dy.$$

(b) $\iiint (x+y+z) dx dy dz$ over the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 0.$ 8,8

UNIT-IV

8. (a) Show that

$$\int_0^1 \frac{x^{m-1} (1-x)^{n-1}}{(a+bx)^{m+n}} dx = \frac{1}{(a+b)^m \cdot a^n} B(m, n).$$

(b) Prove that

$$\int_0^{\pi/2} \frac{\log(1+y \sin^2 x)}{\sin^2 x} dx = \pi [\sqrt{1+y} - 1]; \quad y > -1.$$

8,8

9. (a) Find the equation of the sphere which passes through the origin and meets the axes in A, B and C.

(b) Find the equation of the cone which contains the three co-ordinate axes and the lines through the origin having direction cosines

$l_1, m_1, n_1 : l_2, m_2, n_2$ 8,8