Roll No. [redacted]

Total Pages : 4

GSE/D-19

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ALGEBRA

Paper : BM–III

Time : Three Hours]

[Maximum Marks : 40

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each section.

Compulsory Question

1. (a) Prove that every orthogonal matrix is non-singular. 1½

(b) Prove that the set of vectors (1, 2, 0), (0, 3, 1) and (–1, 0, 1) is linearly independent. 1½

(c) Prove that the quadratic form $q = x_1^2 + x_2^2 + x_3^2$ is positive definite. 2

(d) Show that every identity matrix of order $n \geq 2$ is derogatory. 1½

(e) If α, β, γ are roots of equation $x^3 + px^2 + qx + r = 0$ then find $\sum \frac{1}{\alpha}$. 1½

SECTION–I

2. (a) Prove that every square matrix A can be expressed in one and only one way as P + *i*Q, where P and Q are Hermitian matrices. 4

(b) For the matrix A, find non-singular matrices P and Q such that PAQ is in normal form $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$; hence find the rank of A. 4

3. (a) Prove that the characteristic roots of a real symmetric matrix are all real. 4

(b) Prove that $A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 1 & -6 \\ 3 & 4 & 2 \end{bmatrix}$ satisfies its characteristic equation. Also find its inverse, if it exists. 4

SECTION–II

4. (a) Check whether the following system of equations is consistent or not. Solve if it is consistent.

$4x + 3y + 2z = -7;$

$2x + y - 4z = -1;$

$x + 2y + z = 1.$ 4

(b) Prove that the absolute value of each characteristic root of unitary matrix is unity. 4

5. (a) For what value of λ, the equations

$x + y + z = 1$

$x + 2y + 4z = \lambda$

$x + 4y + 10z = \lambda^2$

have a solution and solve them completely in each case. 4

(b) Diagonalize the quadratic form

$$x_1^2 + 2x_3^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3.$$

Also, find the rank, index, signature and equations of transformation. 4

SECTION–III

6. (a) Find the remainder in the division of $x^3 + 3px + q$ by $(x - a)^2$, and deduce that it has two equal roots if $q^2 + 4p^3 = 0$. 4

(b) If b and c are real and $2 - \sqrt{-3}$ is a root of the equation $x^3 + x^2 + bx + c = 0$, what are the other roots and what is the value of c ? 4

7. (a) Find the condition that the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ is equal to zero. 4

(b) If α, β, γ are the roots of the cubic $x^3 + 3x + 2 = 0$, find the equation whose roots are $(\alpha - \beta)(\alpha - \gamma)$, $(\beta - \gamma)(\beta - \alpha)$, $(\gamma - \alpha)(\gamma - \beta)$. Hence show that the given cubic has two imaginary roots. 4

SECTION–IV

8. (a) Solve the equation $28x^3 - 90x^2 + 1 = 0$ by Cardan's method. 4

(b) Apply Descarte's method to solve the equation $x^4 - 3x^2 - 42x - 40 = 0$. 4

9. (a) Solve $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$ by Ferrari's method. 4

(b) Show that the equation $x^7 + x^4 + 8x + k = 0$ has at least four imaginary roots for all values of k. 4
