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GSE/D-17

CALCULUS

Paper : BM-112

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each section.

Compulsory Question

1. (a) Evaluate $\lim_{x \to 0} \frac{x}{|x|}$. 2

(b) If $y = ae^{mx} + be^{-mx}$, prove that $y_2 - m^2 y = 0$. 2

(c) Define Asymptotes with example. 1

(d) What is a Singular point ? 1

(e) Find the length of a loop of the curve $r = a(\theta^2 - 1)$. 2

SECTION–I

2. (a) Discuss the continuity and differentiability of the function $f(x) = |x - 1| + |x - 2|$ in the interval [0, 3]. 4

(b) If $y = \left[\log\left(x + \sqrt{1 + x^2}\right)\right]^2$, prove that

$$(1 + x^2) y_{n+2} + (2n + 1) x y_{n+1} + n^2 y_n = 0.$$ 4

3. (a) Expand $\sin x$ and $\cos x$ in powers of x, and hence find $\cos 18^\circ$ upto four decimal places. 4

(b) Find the approximate change in the value of $f(x) = 5x^3 - 3x^2 + 7x - 8$, when x changes from 3 to 3.001. 4

SECTION–II

4. (a) Find all the asymptotes of the curve
$$(x + y)^2 (x + y + 2) - x - 9y + 2 = 0.$$ 4

(b) Find the asymptotes of the curve
$$r^2 = a^2(\sec^2 \theta + \operatorname{cosec}^2 \theta).$$ 4

5. (a) Show that in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the radius of curvature at the end of the major axis is equal to the semi-latus rectum of the ellipse. 4

(b) Show that every point, in which the sine curve $y = c \sin \frac{x}{c}$ meets the axis of x, is a point of inflexion. 4

SECTION–III

6. (a) Trace the curve $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$. 4

(b) Obtain a reduction formula for $\int x^n \cos x \, dx$, and hence evaluate $\int x^3 \cos x \, dx$. 4

7. (a) Find that the loop of the curve $x = t^2$, $y = t - \frac{1}{3}t^3$ is of length $4\sqrt{3}$. 4

(b) Find the intrinsic equation of the cardioid $r = a(1 - \cos\theta)$. 4

SECTION–IV

8. (a) Trace the curve $ay^2 = x^2(a - x)$, and show that the area of the loop is $\frac{8}{15}a^2$. 4

(b) Find the area common to the circle $r = a$ and the cardioid $r = a(1 + \cos\theta)$. 4

9. (a) The circle $x^2 + y^2 = a^2$ is revolved about the x-axis. Find the volume of the sphere so formed. 4

(b) Find the surface of the solid generated by the revolution of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis. 4
