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GSE/D-16 **774**

MATHEMATICS

(Solid Geometry)

Paper : BM-113

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all. Select *one* question each from Unit-I to Unit-IV. Question No. 9 (Unit-V) is compulsory.

UNIT–I

1. Show that the conic $8x^2 - 4xy + 5y^2 - 16x - 14y - 17 = 0$ represents ellipse. Find its centre, foci, length of axes, equations of axes, and trace it. 7

2. (a) Prove that the conics $x^2 - y^2 - bx + 2y + 7 = 0$ and $x^2 + 3y^2 - 4x - 6y + 4 = 0$ are confocal. 3½

(b) Show that the equations

$$\frac{l}{r} = 1 + e\cos\theta \text{ and } \frac{l}{r} = -1 + e\cos\theta$$

represent the same conic. 3½

UNIT–II

3. (a) Tangent plane at any point of the sphere $x^2 + y^2 + z^2 = r^2$ meets the axis in A, B, C. Show that the locus of the point of intersection of the planes drawn parallel to the co-ordinate plane through A, B, C is the surface $x^{-2} + y^{-2} + z^{-2} = r^{-2}$. 3½

(b) Find the equation of the right circular cylinder whose guiding circle is

$$x^2 + y^2 + z^2 = 4,\ x + y + z = 3.$$ 3½

4. (a) Find the equation of the right circular cone whose vertex is (0, 0, 0); which passes through (1, 1, 2) and axis $\frac{x}{2} = \frac{y}{-4} = \frac{z}{3}$. 3½

(b) Find the centres of the two spheres, which touch the plane $x + 2y + 2z = 5$ at the point (1, 1, 1) and the sphere

$$x^2 + y^2 + z^2 + 2x + 4y + 6z - 11 = 0.$$ 3½

UNIT–III

5. (a) Find the equations of tangent planes to

$$3x^2 + 2y^2 - 6z^2 = 5$$

which pass through the lines $3x - y - 9z = 0$ and $6x + 3y - 3z - 5 = 0$. 4

(b) Find the centre of the conic given by the equation $x + 6y - 10z + 20 = 0$ and $x^2 + 4y^2 - 5z^2 = 1$. 4

6. (a) Prove that six normals can be drawn from a given point to the ellipsoid. 4

(b) Find the equations of the polar of the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

w.r.t. the conicoid $x^2 - 2y^2 + 3z^2 = 4$ in the symmetrical form. 4

UNIT–IV

7. (a) Show that the plane $2x - 4y - z = 3$ touches the paraboloid $x^2 - 2y^2 = 3z$. Also find the point of contact. 4

(b) Find the equations to the generating lines of the hyperboloid $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$ which pass through the point $\left(2, -1, \frac{4}{3}\right)$ 4

8. Prove that the surface whose equation is

$$16x^2 + 4y^2 + 4z^2 + 4yz - 8zx + 8xy + 4x + 4y - 16z - 24 = 0$$

is an elliptic paraboloid. Find the co-ordinates of its vertex and the equation to its axis. 8

UNIT–V

(Compulsory Question)

9. (a) Find the asymptotes of the hyperbola

$$6x^2 - 7xy - 3y^2 - 2x - 8y - 6 = 0.$$ 2

(b) Find the centre of the conic

$$3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0.$$ 2

(c) Find the centre and radius of the sphere

$$2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0.$$ 2

(d) Find the equations of the tangent planes to the surface $4x^2 - 5y^2 + 7z^2 + 13 = 0$ which are parallel to the plane $4x + 20y - 21z = 0$. 2

(e) Find the equation of the plane which cuts the paraboloid $2x^2 - y^2 = 2z$ in a conic with its centre at the point (2, 3, 4). 2
