

Roll No.
Printed Pages : 3

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GSM / M-18

SPECIAL FUNCTIONS AND INTEGRAL
TRANSFORMS

Paper-BM-242

Time allowed : 3 hours]

[Maximum marks : 40]

Note :- Attempt five questions in all, selecting one question from each unit. Q. No. I is compulsory. All questions carry equal marks.

1. (a) Show that $\frac{d}{dx} [x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$.

(b) Prove that $P_n(x) = \frac{1}{n! 2^n} \frac{d^n}{dx^n} (x^2 - 1)^n$.

(c) Evaluate $\int_0^\pi \frac{\cos x}{x} dx$.

Q1

(d) Solve the integral equation $f(t) = 1 + \int_0^t f(u) \sin(t-u) du$

and verify the solution.

(e) Using Parseval's identity prove that $\int_0^\infty \frac{dx}{(x^2 + 1)^2} = \frac{\pi}{4}$

Unit-I

2. (a) Solve the differential equation in power series:

$$9x(1-x) \frac{d^2y}{dx^2} - 12 \frac{dy}{dx} + 4y = 0$$

Q2

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Turn over

(2)

(b) Find the series solution of the differential equation about $x = 0$:

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = 0$$

3. (a) Prove that $\int_0^r x J_0(ax) dx = \frac{r}{a} J_1(ar)$.

(b) Reduce the differential equation

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (k^2 x^2 - n^2) y = 0$$
 to Bessel's form.

Unit-II

4. (a) Prove that $\int_{-1}^1 P_m(x) P_n(x) dx = 0$ when $m \neq n$ and

$$\int_{-1}^1 [P_n(x)]^2 dx = \frac{2}{2n+1} \text{ when } m = n.$$

(b) Prove that $\int_{-1}^1 x^2 P_n^2(x) dx =$

$$\frac{1}{8(2n-1)} + \frac{2}{4(2n+1)} + \frac{1}{8(2n+3)}$$

5. (a) Prove that $x H_n^1(x) = n H_{n-1}^1(x) + n H_n(x)$.

(b) Prove that $\int_{-\infty}^{\infty} x^2 e^{-x^2} [H_n(x)]^2 dx = \sqrt{\pi} 2^n n! \left(n + \frac{1}{2}\right)$

Unit-III

6. (a) Find the Laplace transform of the function $e^{-t} \cos t \cos 2t$.

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(3)

(b) Find the Laplace transform of $\int_t^\infty \frac{e^{-x}}{x} dx$

7. (a) Use convolution to evaluate $L^{-1}\left[\frac{1}{(s+1)(s+9)^2}\right]$

(b) Solve $t \frac{d^2y}{dt^2} + (1-2t) \frac{dy}{dt} - 2y = 0, y(0) = 1, y'(0) = 2$

Unit-IV

8. (a) If $\int_s^\infty (S) = \frac{s}{1+s^2}$, find $f(x)$

(5)

(b) Find the finite Fourier Sine transform of $\cos ax$ where $0 < x < \pi$.

9. (a) The initial temperature of an infinite bar is given by

$$\theta(x) = \begin{cases} \theta_0 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

Determine the temperature at any point x and at any instant t .

(b) Use finite Fourier transform to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to

the initial conditions.

(i) $u(0,t) = 0$,

(ii) $u(u,t) = 0$

(iii) $u(x,0) = 2x$ where $0 < x < u, t > 0$.