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BAM/A-20 **547**

MATHEMATICS
(Advanced Calculus)
Paper : BM-201

Time : Three Hours]   [Maximum Marks : 45

Note : Attempt *five* questions in all, selecting at least *one* question from each section.

SECTION–I

1. (a) Show that

$$\lim_{n\to\infty}\left[\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdots\cdots\frac{n}{n-1}\right]^{1/n}=1.$$ 4½

(b) Prove that the sequence $<a_n>$ defined by $a_1 = 1$ and $a_n=\sqrt{2+a_{n-1}}$ converges to the positive root of the equation $x^2 - x - 2 = 0$. 4½

2. (a) Use Cauchy's general principle of convergence to show that the sequence $<a_n>$ defined by

$$a_n=1+\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}$$

does not converge. 4½

(b) Test the convergence of the series

$$\sum_{n=1}^{\infty}\left[\sqrt{n^4+1}-\sqrt{n^4-1}\right].$$ 4½

3. (a) Discuss the convergence of the series

$$1+\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+\cdots\cdots$$ 4½

(b) Using Cauchy's Integral test, discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$, $p > 0$. 4½

SECTION–II

4. (a) Prove that if a function f is continuous on a closed interval $[a, b]$, then it is uniformly continuous on $[a, b]$. 4½

(b) Verify Lagrange's Mean value theorem for $f(x) = \sin x$ in $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$. 4½

5. (a) Show that

$$\text{Log}\,(x+h) = \text{Log}\,x + \frac{h}{x} - \frac{h^2}{2x^2} + \cdots + (-1)^{n-1}\frac{h^n}{n(x+\theta h)^n}.$$

4½

(b) Show that the function

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} ; & (x, y) \neq (0, 0) \\ 0 ; & (x, y) = (0, 0) \end{cases}$$

is continuous at (0, 0). 4½

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6. (a) By changing the independent variable u and v to x and y by means of relation $x = u \cos \alpha - v \sin \alpha$, $y = u \sin \alpha + v \cos \alpha$; show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}.$$ 4½

(b) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ using Taylor's theorem upto third degree terms.

SECTION–III

7. (a) Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are connected by the relation $a^n + b^n = c$ and c is constant. 4½

(b) Examine for maximum and minimum values of the function $f(x, y) = xy(a - x - y)$. 4½

8. (a) Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$ using Lagrange's method of multipliers. 4½

(b) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x \right)$. 4½

SECTION–IV

9. (a) Show that $\int_0^{\pi/2} \sqrt{\tan \theta} \, d\theta = \frac{\pi}{\sqrt{2}}$. 4½

(b) Evaluate $\iiint_{x^2+y^2+z^2 \le 1} (x^2 + y^2 + z^2) \, dx \, dy \, dz$. 4½

10. (a) Using Dirichlet's integral, find the volume bounded by the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^4}{c^4} = 1$. 4½

(b) Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx$$

by changing the order of integration. 4½