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BAM/A-20 **548**

MATHEMATICS

(Differential Equations)

Paper : BM-202

Time : Three Hours] [Maximum Marks : 45

Note : Attempt *five* questions in all, selecting at least *one* question from each section.

SECTION–I

1. (a) Solve $2x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (1-x^2)y = 0$ in series. 4½

(b) Use Jacobi's series to show that

$[J_0(x)]^2 + 2[J_1(x)]^2 + 2[J_2(x)]^2 + \ldots\ldots = 1.$ 4½

2. (a) Solve the Legendre equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

by changing it to a hypogeometric equation. 4½

(b) Find the eigen functions of the following Sturm Liouville problem and verify the orthogonality :

$y'' + \lambda y = 0$, $y(0) = 0$ and $y'(l) = 0$. 4½

SECTION–II

3. (a) If $\alpha[f(t)] = F(S)$ then show that

$$\alpha[t^n f(t)] = (-1)^n \frac{d^n}{dS^n}[F(S)],$$

where $n = 1, 2, 3, ...$ 4½

(b) Find the inverse Laplace transform of the function

$$\frac{S}{(S^2 + a^2)^3}.$$ 4½

4. (a) Convert the differential equation

$$f''(t) - 3f'(t) + 2f(t) = 4 \sin t$$

into an integral equation where $f(0) = 1, f'(0) = -2.$ 4½

(b) Solve $t\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + ty = \sin t$ when $y(0) = 1.$ 4½

SECTION–III

5. (a) Eliminate the arbitrary functions and hence obtain the partial differential equation

$$Z = f(x \cos \alpha + y \sin \alpha - at) + \phi(x \cos \alpha + y \sin \alpha + at).$$ 4½

(b) Solve the partial differential equation

$$\left(\frac{b-c}{a}\right)yzp + \left(\frac{c-a}{b}\right)zxq = \frac{a-b}{c}xy.$$ 4½

6. (a) Obtain the complete integral of $z^2(p^2 + q^2) = x^2 + y^2$. 4½

(b) Solve $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y} - x^2y$. 4½

7. (a) Solve

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} + nz = n\left(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right) + x^2 + y^2 + x^3.$$

4½

(b) Solve $r - t\cos^2 x + p\tan x = 0$. 4½

SECTION–IV

8. (a) Prove that variation of the functional $I[y(x)]$ is equal to $\frac{\partial}{\partial \alpha} I[y(x) + \alpha\delta y(x)]$ at $\alpha = 0$, for fixed y and δy and different values of parameter α. 4½

(b) Find the extremals of the functional

$$\int_{\theta_1}^{\theta_2} \sqrt{(r^2 + r'^2)}\, d\theta, \text{ where } r = r(\theta).$$ 4½

9. Light travels in a medium from one point to another so that the time of travel is given by $\int \frac{dS}{V(x, y)}$ where S is the arc length and $V(x, y)$ is the velocity of light in the medium is maximum. Show that the path travelled is given by

$$V^2 \frac{d^2 y}{dx^2} + \left[1 + \left(\frac{dy}{dx}\right)^2\right] \frac{dV}{dy} - \frac{dy}{dx}\left[1 + \left(\frac{dy}{dx}\right)^2\right] \frac{dV}{dx} = 0 .$$ 4½

10. (a) Find the shortest distance between the circle $x^2 + y^2 = 1$ and the straight line $x + y = 4$. 4½

(b) Test for an extremum, the functional

$$I[y(x)] = \int_0^2 (e^{y'} + 3)\, dx, \; y(0) = 0, \; y(2) = 1.$$ 4½