

Roll No.

Total Pages : 4

BAM/A-20

549

MATHEMATICS

(Mechanics)

Paper : BM-203

Time : Three Hours]

[Maximum Marks : 45

Note : Attempt *five* questions in all, selecting at least *one* question from each section. All questions carry equal marks.

SECTION-I

1. (a) Prove that a system of coplanar forces acting in one plane at different points of a rigid body can be reduced to a single force through any given point and a single couple.

(b) A uniform rod AB of weight w and length $2a$ is hinged at A to a fixed point by means of a smooth hinge. It is kept at rest inclined at an angle α to the vertical by means of a force F applied horizontally at B. Find the magnitude and direction of the reaction at the hinge on the rod and show that the magnitude of F is $\frac{1}{2} w \tan \alpha$.
2. Six equal heavy rods, freely hinged at the ends, form a regular hexagon ABCDEF which hung up by the corner A is kept from altering its shape by two light rods BF and CE. Prove that the thrusts in these rods are $\frac{1}{2} 5\sqrt{3} W$ and $\frac{1}{2} \sqrt{3} W$, where W is the weight of each rod.

3. (a) A kite is flowing with 600 metres of string from the hand to the kite and a spring balance held in the hand shows a pull equal to the weight of 100 metres of the string inclined at 30° to the horizon. Find the vertical height of the kite above the hand.
- (b) A telegraph wire has a span of 40 metres. Find the sag in the middle if the tension is not to exceed 400 kg wt., assuming that 1 metre of the wire weighs 1 kg.

SECTION-II

4. (a) Equal forces act along the co-ordinate axes and the straight line $\frac{x - \alpha}{l} = \frac{y - \beta}{m} = \frac{z - \gamma}{n}$. Find the equation of the central axis of the system.
- (b) If P and Q be two non-intersecting forces whose directions are perpendicular, show that the ratio of distance of the central axis from their lines of action are Q^2 to P^2 .
5. (a) Find the null point of the plane $x + y + z = 0$ for the force system (X, Y, Z; L, M, N).
- (b) A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and show that it is unstable.

SECTION-III

6. (a) The position of a moving point at time t is given by $x = a \cos t$ and $y = a \sin t$. Find its path, velocity and acceleration.

- (b) Prove that if the tangential and normal accelerations of a particle describing a plane curve be constant throughout the motion, the angle ψ through which the direction of motion turns is given by

$$\psi = A \log (1 + Bt).$$

7. (a) A particle is describing simple harmonic motion of period T along a straight line. If v be its speed when at a distance x from the mean position and a is the amplitude, show that $v^2 T^2 = 4\pi^2(a^2 - x^2)$.
- (b) An elastic string of natural length $2l$ can just support a certain weight when it is stretched till its length is $3l$. One end of the string is now attached to a point on a smooth horizontal table and the same weight is attached to the other end. Prove that if the weight be pulled to any distance and let go, the string will become slack

after a time $\frac{\pi}{2} \sqrt{\frac{l}{g}}$.

8. (a) A particle moves in a straight line under a retardation kv^2 . If its initial velocity be u , show that the space described in time t is $\frac{1}{k} \log (1 + kut)$.
- (b) If a particle be moving in a medium whose resistance varies as the velocity of the particle, show that equation of trajectory can, by a proper choice of axes, be put in the form $y + ax = b \log x$.

SECTION-IV

9. (a) Show that the orbit described under a central attractive force varying directly as the distance is an ellipse having its centre at the centre of force and conversely.
- (b) Prove that the time taken by earth to travel over half its orbit remote from the sun, separated by minor axis is two days more than half the year; the eccentricity of orbit being $\frac{1}{60}$.
10. (a) A particle describes the equiangular spiral $r = ae^{\theta \cot \alpha}$ under a force to the pole. Find the law of force.
- (b) A particle moves on a smooth sphere under no forces except the pressure of the surface. Show that its path is given by the equation $\cot \theta = \cot \beta \cos \phi$, where θ and ϕ are its angular co-ordinates.
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