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BAQ/A-20 **583**

MATHEMATICS

(Abstract Algebra)

Paper : BM-302

Time : Three Hours] [Maximum Marks : 45

Note : Attempt *five* questions in all, selecting at least *one* question from each section.

SECTION–I

1. (a) Prove that the centre Z of a group G is a normal subgroup of G. 5

(b) Show that every subgroup of an abelian group is normal. 4

2. (a) If $O(G) = p^n$ where p is a prime number, then the centre $Z(G) \neq \{e\}$. 5

(b) Prove that the normalizer of a group G, i.e., $N(a)$ is a subgroup of G. 4

3. State and prove class equation of a finite group G. 9

SECTION–II

4. (a) Prove that the set Z of all integers is a ring with respect to addition and multiplication of integers as the two ring compositions. 5

(b) Prove that every field is an integral domain. 4

5. (a) Prove that the intersection of any two ideals of a ring is again an ideal. 5

(b) R is an integral domain, iff R[x] is an integral domain. 4

SECTION–III

6. (a) The necessary and sufficient condition for non-empty subset W of a vector space V(F) to be a subspace of V is that $au + v \in$ W for all $u, v \in$ W and $a \in$ F. 5

(b) Show that the set of vectors (1, 2, 0), (0, 3, 1) and (1, 0, 1) of $V_3(Q)$ is linearly dependent, where Q is the field of rational numbers. 4

7. (a) Show that the function T : $R^2 \to R^3$ defined by $T(x, y) = (2x, x - y, x + y)$ is a linear transformation and is one-one but not onto. 5

(b) Prove that a linear transformation T : U $\to$ V carries L.D. vectors of U into L.D vectors of V. 4

8. If T : U $\to$ V is a linear transformation then

Rank T + Nullity T = dim U

i.e., $\delta(T) + \mu(T) = \dim U$. 9

SECTION–IV

9. Let V be an inner product space. Then for arbitrary vectors $u, v \in$ V, prove that $|(u, v)| \le \| u \| \| v \|$. 9

10. Show that every finite dimensional inner product space has an orthonormal basis. 9