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BT-1/D-18 **31047**

CALCULUS AND LINEAR ALGEBRA

Paper : BS-133A

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) What is Gamma Function? Explain its reduction formula.

(b) Find the volume formed by the revolution of loop of the curve $y^2(a + x) = x^2(3a - x)$ about the x-axis.

2. (a) State and prove Rolle's Theorem. If $f(x) = x(x-2)e^{\frac{3x}{4}}$ in (0, 2) find c.

(b) Evaluate $\lim_{x \to 0} \frac{\left(1+\frac{1}{x}\right)^{\frac{1}{x}} - e}{x}$.

UNIT–II

3. (a) Reduce the following matrix into normal form and hence find its rank :

$$\begin{bmatrix} 1 & -1 & -2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}.$$

(b) Are the following vectors linearly independent ? If so, find the relation between them : (1, 1, 1, 3); (1, 2, 3, 4); (2, 3, 4, 9).

4. (a) Solve the following simultaneous equations by matrix inversion method.

$x + y + z = 3$, $x + 2y + 3z = 4$, $2x + 3y + 4z = 7$.

(b) Expressing matrix form, solve the following system of simultaneous equations using Gauss Jordan method.

$2x_1 + x_2 + 2x_3 + x_4 = 6$, $6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$, $4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$ and $2x_1 + 2x_2 - x_3 + x_4 = 10$.

UNIT–III

5. (a) Why isn't R^2 a subspace of R^3.

(b) If u, v, w are independent vectors, show that the sums $v_1 = w_2 + w_3$, $v_2 = w_1 + w_3$ and $v_3 = w_1 + w_2$ are independent. (Write $c_1v_1 + c_2v_2 + c_3v_3 = 0$ in terms of the w's. Find and solve equations for the c's.

6. (a) Find the range and kernel (those are the new words for the column space and nullspace) of T when $T(v_1, v_2) = (v_1, v_2)$.

   (b) Suppose $T(v) = v$, except that $T(0, v_2) = (0, 0)$. Show that this transformation satisfies $T(cv) = cT(v)$ but not $T(u + v) = T(u) + T(v)$.

UNIT–IV

7. (a) Solve the system of equations

$$x_1 + 2x_3 - 2x_4 = 0, \quad 2x_1 - x_2 - x_4 = 0,$$
$$x_1 + 2x_3 - x_4 = 0, \quad 4x_1 - x_2 + 3x_3 - x_4 = 0.$$

   (b) (i) Modulus of each of the characteristic roots of a unitary matrix is unitary.

   (ii) Any square matrix can be expressed uniquely as a sum of Hermitian and Skew-Hermitian.

8. (a) Reduce the matrix $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form and hence find its canonical form.

   (b) Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence compute A^{-1}.