

BT-I/D-20

41046

CALCULUS & LINEAR ALGEBRA

Paper : BS-133A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT-I

1. (a) Prove that $\int_m^{m+\frac{1}{2}} \sqrt{x} dx = \frac{\sqrt{\pi}}{2^{2m-1}} \int_{2m}^{2m+1} \sqrt{x} dx$ where \int

represents the gamma function and $\sqrt{\quad}$ is the square root function.

(b) Find the volume of a sphere of radius a .

2. (a) Show that $\lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{x}} - e + \frac{ex}{2}}{x^2} = \frac{11e}{24}$.

(b) State Cauchy Mean value theorem and verify Cauchy Mean value theorem for the functions e^x and e^{-x} in the interval (a, b) .

3. (a) If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, and I is identity matrix of

order 3, evaluate. $A^3 - 4A^2 - 3A + 11I$.

(b) Solve the following system of equations using Cramer's rule.

$$x + 3y + 6z = 2$$

$$3x - y + 4z = 9$$

$$x - 4y + 2z = 7.$$

4. (a) Find the rank of the matrix

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}.$$

(b) Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ and

verify $A^{-1}.A = I$ where I is identity matrix of order 3.

UNIT-III

5. (a) For what value of k will the vector $u = (1, k, 5)$ in $V_3(\mathbb{R})$ be a linear combination of vectors $v = (1, -3, 2)$ and $w = (2, -1, 1)$.
- (b) Show that the set $\{(2, 4, -3), (0, 1, 1), (0, 1, -1)\}$ forms a basis of \mathbb{R}^3 .
6. (a) Show that the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(x, y) = (x, x - y, x + y)$ is a linear transformation.
- (b) For the linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ such that $T(x, y) = (x - y, y - x, -x)$, find a basis and dimension of its range space and its null space. Also verify, that $\text{rank}(T) + \text{nullity}(T) = \dim \mathbb{R}^2$.

UNIT-IV

7. (a) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

- (b) Let V be an inner product space. Show that

$$\|u + v\| \leq \|u\| + \|v\|.$$

8. (a) Define Orthogonal Matrix; also show that the matrix

$$\begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

is orthogonal.

- (b) If A is a square matrix, show that (i) $A + A'$ is symmetric, (ii) $A - A'$ is skew-symmetric, where A' is the transpose of A .

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