Roll No. ..... Total Pages : 3

BT-I/D-17 **31001**

MATHEMATICS–I

Paper : Math-101(E)

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *five* questions in all selecting at least *one* question from each unit.

UNIT–I

1. (a) Using Taylor's series, compute the value of sin 31° to four decimal places. 10

(b) Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=1$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$. 10

2. (a) Find the asymptotes of the curve

$$6x^2 + xy - 2y^2 + x + 2y + 1 = 0.$$ 10

(b) Trace the curve $y = (x - 2)(x + 1)^2$. 10

UNIT–II

3. (a) If $u = \log (x^3 + y^3 + z^3 - 3xyz)$, show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}.$$ 10

(b) If $z = f(x, y)$ and $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}.$$ 10

4. (a) If $u = x\,\phi(y/x) + \psi(y/x)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0.$$ 10

(b) Examine for extreme values

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2.$$ 10

UNIT–III

5. (a) Evaluate $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} x\,dx\,dy$ by changing the order of integration. 10

(b) Evaluate $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}\,dz\,dy\,dx$. 10

6. (a) Show that the volume of the solid obtained by the revolution of the curve $a^2y^2 = x^2(a^2 - x^2)$ about the x-axis is $\frac{4a^3}{15}$. 10

(b) Derive the relation between Beta and Gamma functions. 10

UNIT–IV

7. (a) Find the values of λ and μ such that the surfaces
$$\lambda x^2 - \mu yz = (\lambda + 2)x \text{ and } 4x^2y + z^3 = 4,$$
may intersect orthogonally at the point $(1, -1, 2)$. 10

(b) Define divergence of a vector point function, and discuss its physical significance. 10

8. (a) Evaluate $\iint_S \vec{F}.\bar{n}\, dS$ where $\vec{F} = 18z\,\hat{i} - 12\hat{j} + 3y\,\hat{k}$ and S is the surface of the plane $2x + 3y + 6z = 12$, in the first octant. 10

(b) Use divergence theorem to evaluate $\iint_S \vec{F}.d\vec{S}$ where $\vec{F} = x^3\,\hat{i} + y^3\hat{j} + z^3\,\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. 10
