Roll No. ..... Total No. of Pages : 2

BT-1/D11 7501

Mathematics-I

Paper : MATH-101 E

Time : Three Hours] [Maximum Marks : 100

Note :– Attempt **FIVE** questions, selecting at least **ONE** question from each Unit.

UNIT–I

1. (a) Expand $\tan\left(\frac{\pi}{4}+x\right)$, by Taylore's series and hence find tan (46°5°) correct to four decimal places.

   (b) Show that the radius of curvature at an end of the major axis of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the semi-latus rectum.

2. (a) Find the asymptotes of :
   $(x + y)^2 (x + 2y + 2) = (x + 9y - 2)$.

   (b) Trace the curve :
   $y^2(x - a) = x^2(x + a)$.

UNIT–II

3. (a) If $z = x\,\phi\left(\frac{y}{x}\right)+\psi\left(\frac{x}{y}\right)$, prove that, by using Euler's Theorem,
   $$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=0.$$

   (b) Transform the equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$ into polar coordinates.

4. (a) In estimating the cost of a pile of bricks measured as 6m × 50 m × 4m. The tape is streched 1% beyond the standard length. If the count is 12 bricks in $1m^3$ and bricks cost Rs. 600/- per thousand. Find the approximate error in cost.

   (b) Evaluate $\int_0^{\pi}\log(1+a\cos x)\,dx$, using the method of differentiation under the sign of integration.

UNIT–III

5. (a) Change the order of integration and then evaluate $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$.

   (b) Find, by triple triangle, the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

6. (a) Calculate, by double integral, the volume generated by the revolution of the cardioid $r = a(1 - \cos\theta)$ about its axis.

   (b) Express $\int_0^1 x^m(1-x^n)^p\,dx$, in Terms of gamma function and evaluate $\int_0^1 x^4(1-x^4)^{10}\,dx$.

UNIT–IV

7. (a) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1,3,–1). http://www.kuonline.in

   (b) Prove that curl :
   $$(\bar{F}\times\bar{G})=\bar{F}\cdot\operatorname{div}\bar{G}-\bar{G}\operatorname{div}\bar{F}+(\bar{G}\cdot\nabla)\bar{F}-(\bar{F}\cdot\nabla)\bar{G}.$$

8. (a) Using Green's Theorem, evaluate :-
   $\oint_C[(y-\sin x)\,dx+\cos x\,dy]$, where C is the plane triangle enclosed by the line $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$.

   (b) Evaluate $\iint(x\,dy\,dz + y\,dz\,dx + z\,dx\,dy)$ of the surface of the sphere of radius a.