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BT-I/D-19 **31040**

MULTIVARIABLE CALCULUS AND LINEAR ALGEBRA

Paper : BS-135A

Opt. (II)

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. All questions carry equal marks.

UNIT–I

1. (a) Prove that $\int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{1}{4\sqrt{2}} \cdot \frac{\Gamma\left(\frac{1}{4}\right) \cdot \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{3}{4}\right)}$. 7½

(b) Find the surface generated by revolving the portion of the curve $y^2 = 4 + x$ cut-off by the straight line $x = 2$, about the x-axis. 7½

2. (a) Prove that between any two roots of $e^x \sin x = 1$, there is at least one root of $e^x \cos x + 1 = 0$. 7½

(b) Evaluate : $\underset{x\to 0}{\text{Lt}} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$. 7½

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UNIT–II

3. (a) Test for convergence of $\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+3^2} + ...$ 7½

(b) Test for convergence of the series

$$x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{x^5}{5} + ... \infty, x > 0.$$ 7½

4. (a) Find the Fourier series of $f(x) = \sqrt{1 - \cos x}$, $0 < x < 2\pi$. Deduce that $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$. 7½

(b) Expand $f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1 \end{cases}$, as a Fourier series of sine and $f(x + 2) = f(x)$. 7½

UNIT–III

5. (a) Apply Taylor's theorem to express $\sqrt{x}$ about the point $x = 1$, upto third degree. 7½

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(b) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{zx}\right)$, then find the value of

$$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z}.$$ 7½

6. (a) If $u = x^2 - 2y$; $v = x + y + z$; $w = x - 2y + 3z$, then find

$$\frac{\partial(u, v, w)}{\partial(x, y, z)}.$$ 7½

(b) The pressure P at any point (x, y, z) in space is $P = 400\, xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$. 7½

UNIT–IV

7. (a) Find the matrix inverse by Gauss Jordan method :

$$\begin{bmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{bmatrix}.$$ 7½

(b) Test for consistency and solve the following system of linear equations :

$$4x_1 - x_2 = 12$$

$$-x_1 + 5x_2 - 2x_3 = 0$$

$$-2x_2 + 4x_3 = -8.$$ 7½

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8. (a) Examine the following system of vectors for linear dependence. If dependent, find the relation between them.

$$X = (1, -1, 1);\ Y = (2, 1, 1);\ Z = (3, 0, 2).$$ 7½

(b) Verify the Cayley-Hamilton theorem for the following matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and use the theorem to find A^{-1}. 7½

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