

Roll No. ....

Total Pages : 04

**BT-2/M-17****8230****MATHEMATICS****AS-104-N****Applied Mathematics-II**

Time : Three Hours]

[Maximum Marks : 75]

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

**Unit I**

1. (a) Solve the equation  $x^4 + 8x^3 + 14x^2 - 8x - 15 = 0$ , given that the roots are in AP.

- (b) Find the equation whose roots are  $\alpha + \frac{1}{\beta\gamma}$ ,  $\beta + \frac{1}{\alpha\gamma}$

$\gamma + \frac{1}{\alpha\beta}$ , where  $\alpha, \beta, \gamma$  are the roots of the cubic

equation  $x^3 + \frac{a}{2}x^2 - \frac{b}{2}x + c = 0$ .

2. (a) Show that :

$$\beta(p, q) = \int_0^\infty \frac{y^{q-1}}{(1+y)^{p+q}} dy = \int_0^1 \frac{x^{p-1} + x^{q-1}}{(1+x)^{p+q}} dx$$

- (b) By using Leibnitz Rule 1 solve :

$$\int_0^\infty e^{-x} \frac{\sin ax}{x} dx$$

**Unit II**

3. (a) Find the Laplace transform of the functions :

(i)  $|t-1| + |t+1|$  where  $t \geq 0$

(ii)  $t \int_0^t e^{-4t} \cdot \cos 3tdt$ .

- (b) Find the inverse Laplace transform of the functions :

(i)  $\tan^{-1}\left(\frac{a}{s}\right)$

(ii)  $\frac{7s-11}{(s+1)(s-2)^2}$ .

4. (a) If  $u$  defines unit step function, then prove that  $L[f(t-a)u(t-a)] = e^{-as} \bar{f}(s)$ , using this property

find  $L^{-1}\left[\frac{se^{-s/2}}{s^2 + \pi^2}\right]$ .

- (b) Using Laplace transform, solve

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 8y = \cos 2x, \text{ given that } y = 2 \text{ and } \frac{dy}{dx} = 1 \text{ at } x = 0.$$

**Unit III**

5. (a) Solve the equation :

$$\left( xy^2 - e^{\frac{1}{x^3}} \right) dx - x^2 y dy = 0$$

- (b) Show that the family of orthogonal trajectories of  $x^2 + y^2 = 2ax$  represents a family of circles  $x^2 + y^2 = cy$ ,  $c$  being a parameter.
6. (a) By method of variation of parameter, solve :

$$\frac{d^2y}{dx^2} + y = \sec x \tan x.$$

- (b) Solve the set of simultaneous equations :

$$\frac{d^2y}{dt^2} + y = \sin t, \quad \frac{d^2y}{dt^2} + x = \cos t.$$

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**Unit IV**

7. (a) Find the directional derivative of  $f(x, y, z) = 2x^2 + 3y^2 + z^2$  at the point P(2, 1, 3) in the direction of the vector  $\hat{a} = \hat{i} - 2\hat{k}$ .
- (b) (i) If  $u = x^2 + y^2 + z^2$  and  $\nabla = x\hat{i} + y\hat{j} + z\hat{k}$ , show that  $\operatorname{div}(4\nabla) = 5u$ .

(ii) If  $\mathbf{R} = x\hat{i} + y\hat{j} + z\hat{k}$  and  $r = \sqrt{x^2 + y^2 + z^2} \neq 0$ , then show that  $\nabla \left( \nabla \cdot \frac{\mathbf{R}}{r} \right) = -\frac{2}{r^3} \mathbf{R}$ .

8. (a) Evaluate by Green's theorem

$\int_C (x^2 - \cosh y) dx + (y + \sin x) dy$ , where C is the rectangle with vertices (0, 0), ( $\pi$ , 0), ( $\pi$ , 1) and (0, 1).

- (b) Evaluate  $\int_C (xy dx + xy^2 dy)$  by using Stoke's theorem, where C is the square in the xy-plane with vertices (1, 0), (-1, 0), (0, 1) and (0, -1).

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