

Roll No.

Total Pages : 04

BT-2/M-17 **8201**MATHEMATICS-II
MATH-102-E
(2006 to Onwards)

Time : Three Hours]

[Maximum Marks : 100]

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Using Gauss Jordan method, find the inverse of the matrix A : **10**

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$$

- (b) Show that if $\lambda \neq -5$, the system of equations $3x + y + 4z = 3$, $x + 2y - 3z = -2$, $6x + 5y + \lambda z = -3$ have a unique solution. If $\lambda = -5$, show that the equations are consistent. Determine the solution in each case. **10**

2. Find the eigen values and the corresponding eigen vector of the following matrix :

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Show that the equation is satisfied by A and hence obtain the inverse of the given matrix. **20**

Unit II

3. (a) Solve : **10**

$$(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$$
- (b) Find the orthogonal trajectories of the family of confocal and coaxial parabola $r = 2a/(1 + \cos \theta)$. **10**

4. (a) Find the complete solution of : **10**

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

- (b) Solve by the method of variation of parameter : **10**

$$y'' - 2y' + y = e^x \log x.$$

Unit III

5. (a) (i) Evaluate : **5**

$$\int_0^t \frac{e^t \sin t}{t} dt$$

(ii) Evaluate :

5

$$\mathcal{L}^{-1}\left[\frac{e^{-s}}{(s+1)^3}\right]$$

(b) Evaluate :

10

$$\mathcal{L}^{-1}\left[\frac{s^3}{s^4 - a^4}\right]$$

6. (a) State convolution theorem and apply convolution theorem to evaluate : 10

$$\mathcal{L}^{-1}\left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right]$$

(b) Solve the transform method :

$$ty'' + 2y' + ty = \cos t \text{ given that } y(0) = 1. \quad 10$$

Unit IV

7. (a) (i) Form the P.D.E. from $F[x^2 + y^2, z - xy]$. 5

(ii) Solve :

$$(x^2 - y^2 - z^2)p + 2xyq = 2xz. \quad 5$$

- (b) Solve by Charpit's method : 10

$$pxy + pq + qy = yz$$

8. (a) By the method of separation of variables, solve :

$$4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$$

given $u = 3e^{-y} - e^{-5y}$ when $x = 0$. 10

- (b) A tightly stretched flexible string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$, the string is given a shape defined by $F(x) = \mu x(l - x)$, where μ is a constant and then released. Find the displacement of any point x of the string at any time $t > 0$. 10