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BT-3 / D-17

DISCRETE STRUCTURES

Paper–CSE–201N

Time allowed : 3 hours] *[Maximum marks : 75*

Note : *Attempt any five questions. All questions carry equal marks.*

1. (a) Let A, B and C be sets such that

$(A \cap B \cap C) = \phi, (A \cap B) = \phi, (A \cap C)\phi,$

and $(B \cap C) = \phi$. Draw the corresponding Venn diagram. 8

(b) Let A and B be two arbitrary sets. Show that $P(A \cap B) = P(A) \cap P(B)$. Give a counter example. 7

2. (a) Formulate and prove by induction a general formula stemming from the observations that.

$1^3 = 1$

$2^3 = 3 + 5$

$3^3 = 7 + 9 + 11$

$4^3 = 13 + 15 + 17 + 19$ 8

(b) Construct the truth tables for following statements:

(i) $P \longrightarrow P$

(ii) $(P \longrightarrow P) \vee (P \longrightarrow \tilde{P})$

(iii) $(P \longrightarrow P) \longrightarrow (P \longrightarrow \tilde{P})$ 7

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3. (a) Let R be a binary relation on the set of all positive integers such that R = { (a,b) | a – b is an odd positive integer} Is R reflexive? Symmetric? Antisymmetric? Transitive ? An equivalence relation? A partial ordering relation? Discuss. 8

(b) Let R be a symmetric and transitive relation on a set A. Show that if for every 'a' in A there exists 'b' in A such that (a,b) is in R, then R is an equivalence relation. 7

4. Let $(A, \leq)$ be a partially ordered set. Let $\leq_R$ be a binary relation on A such that for 'a' and 'b' in A, a $\leq_R$ b if and only if b $\leq$ a.

(a) Show that $\leq_R$ is a partial ordering relation.

(b) Show that if $(A, \leq)$ is a lattice, then $(A, \leq_R)$ is also a lattice. 15

5. We are given a red box, a blue box, and a green box. We are also given 10 red balls, 10 blue balls, and 10 green balls. Balls of the same color are considered identical. Consider the following constraints:

(a) No box contains a ball that has the same color as the box.

(b) No box is empty.

Determine the number of ways in which we can put the 30 balls in to boxes so that:

(i) No constraint has to be satisfied ; that is, every combination is permitted.

(ii) Constraint 1 is satisfied.

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(iii) Constraint 2 is satisfied.

(iv) Constraints 1 and 2 are satisfied. 15

6. (a) Determine the generating function of the numeric function a_r, where. 8

$$a_r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases}$$

(b) Solve the following recurrence relations:

(i) $a_r - 7a_{r-1} + 10\, a_{r-2} = 3$. Given that $a_0 = 0$ and $a_1 = 1$

(ii) $a_r + a_{r-1} + a_{r-2} = 0$. Given that $a_0 = 0$ and $a_1 = 2$. 7

7. (a) Let (A, ∗) be an algebraic system such that for all a, b, c, d in A $a * a = a$

$(a * b) * (c * d) = (a * c) * (b * d)$

Show that

$a * (b * c) = (a * b) * (a * c)$ 8

(b) Let (A, ∗) be a semigroup. Show that, if A is a finite set, there exists 'a' in A such that $a * a = a$ 7

8. (a) Let (A, ∗) be a monoid such that for every x in A, $x * x = e$, where e is the identity element. Show that (A, ∗) is an abelian group. Also show that any subgroup of a cyclic group is cyclic. 8

(b) Let (A, ∗) be a semigroup. Show that, for a, b, c in A, if $a * c = c * a$ and $b * c = c * b$, then $(a * b) * c = c * (a * b)$. 7

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