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CMDQ/M-20 **5524**

ADVANCED DISCRETE MATHEMATICS
MSM-406

Time : Three Hours] [Maximum Marks : 70

Note : Attempt *Five* questions in all, selecting at least *two* questions from each Section. All questions carry equal marks.

Section I

1. (a) Let L and L' be lattices. Prove that a 1–1 and onto map $f : L \to L'$ is an isomorphism of lattices iff both f and f^{-1} are order perserving maps.

(b) Prove that the lattice of all subspaces of a vector space V over a field F is a modulator lattice, but need not be distributive.

2. (a) Let L be a modular lattice and $a, b \in L$ be such that $a \geq b$. Prove that any two finite chains connecting a and b have isomorphic requirements.

(b) Let $a_1, a_2, ..., a_m, a_{m+1}$ be elements of a modular lattice L. Prove that if $a_1, a_2, ..., a_m$ are independent and $a_{m+1} \wedge (a_1 \vee a_2 \vee .... \vee a_m) = 0$, then $a_1, a_2, ..., a_m, a_{m+1}$ are independent.

3. (a) Prove that in a complemented modular lattice either of the chain conditions implies the other.

(b) Prove that a Boolean algebra defines a Boolean ring with unity.

4. (a) Prove that a finite Boolean algebra is isomorphic to the Boolean algebra of the power set of a set X.

(b) Draw a simplified switching circuit of the switching function :

$$F(a, b, c) = a.c + b.(b' + c).(a' + b.c')$$

Section II

5. (a) Let G = (V, E) be a simple (n, m) graph of k components. Prove that :

$$m \leq \frac{(n-k)(n-k+1)}{2}$$

(b) Prove that a connected graph G remains connected after removing an edge e from G iff e is in some circuit in G.

6. (a) Prove that a connected graph G is a Euler graph iff degree of each vertex of G is even.

(b) Prove that a connected graph of n vertices and $n - 1$ edges is a tree.

7. (a) Let S_1 and S_2 be two distinct cut-sets of a connected graph G. Prove that $S_1 \oplus S_2$ is the union of edge disjoint cut-sets.

(b) Prove that the Kuratowski's second graph $k_{3,3}$ is non-planar.

8. (a) Prove that the dimension of the circuit subspace corresponding to a connected graph G is the nullity of G.

(b) Prove that the rank of the incidence matrix A(G) of a connected graph G is the rank of G.