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CMDQ/M-20 **5526**

ALGEBRAIC NUMBER THEORY
MSM-408

Time : Three Hours] [Maximum Marks : 70

Note : Attempt *Five* questions in all, selecting at least *two* questions from each Section.

Section I

1. State and prove Liouville theorem. Using this theorem prove that : **14**

$$\sum_{n=0}^{\infty} \frac{1}{10^{\lfloor n}}$$

is transcendental.

2. Let $m \in \mathbf{Z}$ and α be an algebraic integer, let $f(x)$ be the minimal polynomial of α. Show that : **14**

$$d_{k/\mathbf{Q}}(\alpha + m) = (-1)^{{}^nC_2} \prod_{i=1}^{n} f'\left(\alpha^{(i)}\right)$$

3. If $\mathbf{Q} \subseteq \mathrm{K} \subseteq \mathrm{L}$ and K, L are algebraic number fields, prove that $d_{\mathrm{K}}/d_{\mathrm{L}}$. **14**

4. Show that every non-zero prime ideal of O_k is maximal and every unique factorization domain is integrally closed. **14**

Section II

5. Determine the prime ideal factorization of (7), (29) and (31) in $k = \mathbf{Q}\left(2^{1/3}\right)$. **14**

6. Show that the equation : **14**

$$x^2 + 5 = y^3$$

has no integral solution.

7. Show that number of quadratic residues mod p is equal to the number of quadratic non-residues mod p. Hence prove that $\sum_{a=1}^{p-1}\left(\frac{a}{p}\right) = 0$ for any fixed prime p. **14**

8. If p is a prime congruent to 13 or 17 (mod 20), show that $x^4 + py^4 = 25z^4$ has no solution in integers. **14**