**Roll No. ..........................** **Total Pages : 05**

**CMDQ/M-20** **5651**

# CONDENSED MATTER PHYSICS-II

## Phy-403-A

Time : Three Hours] [Maximum Marks : 60

**Note :** Attempt *Five* questions in all. Q. No. **1** is compulsory. Attempt the remaining *four* questions by selecting *one* question from each Unit.

**1.** (a) Explain that the electrical current density for a partially filled band can be described as a current density of positively charged particles (i.e., holes), assigned to the unoccupied states of the band. **3**

(b) What are the van Hove singularities ? How do these reflect in the optical properties of one-dimensional systems ? **3**

(c) Hartree and Hartree-Fock methods provide a way to treating the effect of electron-electron interactions on physical propeties of a solid. Explain the main assumptions underlying these methods. Write at least *two* drawbacks for each of the two methods. **3**

(d) Define field operators. Deduce their commutation/anti-commutation relations for bosons/fermions. Write Hamiltonian H for a system of bosons/fermions in terms of field operators. **3**

**Unit I**

**2.** (a) Define the electron effective mass. Illustrate graphically its dependence on the wave vector $k$ for a one-dimensional band structure in the case of strong and weak curvature of band. **4**

(b) Explain the one-electron approximation for a crystalline solid, and use it to describe different sources of scattering of electrons. In principle, one can go beyond the one-electron approximation by including the electron-electron interactions. Show that the contribution of these interactions is in fact relatively quite small. **8**

**3.** (a) Setup the Boltzmann equation for electric current in a solid, and obtain its linearized form for the determination of non-equilibrium distribution function. **8**

(b) Continuing with part (a), show the effect of weak electric field on the Fermi sphere and the Fermi distribution function in the vicinity of the Fermi energy. **4**

## Unit II

**4.** (a) What are nanostructures ? Explain that XRD is of limited use in determining the structure of a nanomaterial. Discuss the principle of scanning tunneling microscopy (STM) for finding the structure of such a material. **6**

(b) What are quantum dots ? Show that the optical properties of a small metallic quantum dot are typically dominated by the surface plasmon resonance. **6**

**5.** (a) Consider a one-dimensional channel connecting two larger reservoirs with a voltage V between them. Find the electrical conductance to obtain the Landauer Formula. Further, show that the resistance of the device can be expressed as the sum of the quantized contact resistance and the resistance due to scattering from barriers in the channel. **6**

(b) For electrons in a square GaAs wire of width 20 nm, find the linear electron density at which the $n_x = 2$, $n_y = 2$, sub-band is first populated in equilibrium at T = 0 K. Assume an infinite confining potential at the wire boundary. **6**

## Unit III

**6.** (a) Apply the Hartree-Fock theory to homogeneous electron gas in Jellium model to calculate the effect of *e-e* interaction on the single-electron energy. Estimate the band width and compare it with Hartree result and photoelectron emission experiment. **9**

(b) What do you understand by the terms : "Exchange energy" and "Correlation energy" ? **3**

**7.** (a) Show that the inverse Fourier transform of screened Coulomb potential $\phi(q) = \dfrac{4\pi Q}{q^2 + k_0^2}$ is given by $\phi(r) = \dfrac{Q}{r}\exp\left(-k_0 r\right)$. **4**

(b) Consider that a positively charged particle is placed rigidly at a given position in a metal. Use the Lindhard approach to determine the induced charged density $\rho^{\text{ind}}(q)$ and the dielectric function $\varepsilon(q)$ . Under what condition the Lindhard result reduces to the Thomas-Fermi result ? **8**

## Unit IV

**8.** Consider a macroscopic homogeneous system of N identical interacting bosons. Starting with the configuration space Schrödinger wave equation, give main steps of deriving the second-quantized wave equation. **12**

**9.** (a) Describe the Jellium model of metal. Show that its Hamiltonian H can be expressed as : $H = -\frac{1}{2}e^2 N^2 V^{-1} 4\pi\mu^{-2} + H_{el}$. What is the significance of the parameter $\mu$ in this expression ? **8**

(b) Define $r_s$ parameter. Explain its physical significance. Calculate the kinetic energy of non-interacting electrons in ground state in terms of $r_s$. **4**