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MDQ/M-20 **4235**

INTEGRAL EQUATIONS AND BOUNDARY VALUE PROBLEMS

Paper X (MM-505) (Opt. i)

Time : Three Hours] [Maximum Marks : 80

Note : Question paper is divided into four Sections. The candidates are requested to attempt *five* questions, selecting at least *one* question from each Section.

Section I

1. (a) Define a L_2-function i.e. a square integrable function. Define regularity conditions. **8**

(b) Solve the integral equation : **8**

$$g(s) = f(s) + \lambda \int_0^1 (s + t)\, g(t)\, dt$$

and find the eigenvalues.

2. (a) Solve the homogeneous Fredholm integral equation : **8**

$$g(s) = \lambda \int_0^1 e^s e^t \; g(t)\, dt$$

(b) Solve the integral equation : **8**

$$g(s) = f(s) + \lambda \int_0^1 (1 - 3st)\, g(t)\, dt$$

3. (a) Find the Neumann series for the solution of the integral equation : **8**

$$g(s) = (1 + s) + \lambda \int_0^s (s - t)\, g(t)\, dt$$

(b) Explain method of successive approximations. **8**

Section II

4. State and prove Riesz-Fisher Theorem. **16**

5. Discuss the solution of the Cauchy-type singular integral equation. **16**

Section III

6. Discuss initial value problems and Boundary Value Problems. **16**

7. Discuss representation formulas for the solutions of the Laplace and Poisson equations. **16**

8. State and prove Poisson's integral formula. **16**

Section IV

9. (a) Find the resolvent of the integral equation : **8**

$$g(s) = f(s) + \int_0^s (s-t)\, g(t)\, dt$$

(b) Solve the integral equation : **8**

$$\sin s = \left(\frac{1}{\pi}\right) \int_{-\infty}^{\infty} \left[g(t)/(t-s) \right] dt$$

10. Explain generalized three-part boundary value problem. **16**